

Relations & Functions

1. A function $f: \mathbb{R}_+ \rightarrow \mathbb{R}$ (where \mathbb{R}_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is: (2024)

- (A) one-one but not onto
- (B) onto but not one-one
- (C) both one-one and onto
- (D) neither one-one nor onto

Ans. (A) one-one but not onto

2. A relation R on set $A = \{1, 2, 3, 4, 5\}$ is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive. (2024)

Ans.

(a) Reflexive:

$\because |x^2 - x^2| < 8 \forall x \in A \Rightarrow (x, x) \in R \therefore R$ is reflexive.

(b) Symmetric:

Let $(x, y) \in R$ for some $x, y \in A$

$\therefore |x^2 - y^2| < 8 \Rightarrow |y^2 - x^2| < 8 \Rightarrow (y, x) \in R$

Hence R is symmetric.

(c) Transitive:

$(1, 2), (2, 3) \in R$ as $|1^2 - 2^2| < 8, |2^2 - 3^2| < 8$ respectively

But $|1^2 - 3^2| \not< 8 \Rightarrow (1, 3) \notin R$

Hence R is not transitive

3. A function f is defined from $\mathbb{R} \rightarrow \mathbb{R}$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function $f(x)$. Hence, check whether function $f(x)$ is one-one and onto or not. (2024)

Ans. $f(x) = ax + b$

Solving $a + b = 1$ and $2a + b = 3$ to get $a = 2, b = -1$

$f(x) = 2x - 1$

Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in \mathbb{R}$

$2x_1 - 1 = 2x_2 - 1 \Rightarrow x_1 = x_2$

Hence f is one - one.

Let $y = 2x - 1, y \in \mathbf{R}$ (Codomain)

$\Rightarrow x = \frac{y+1}{2} \in \mathbf{R}$ (domain)

Also, $f(x) = f(\frac{y+1}{2}) = y$

\therefore f is onto.

4.

If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

(2024)

Ans.

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

Put $x = \sin \theta, y = \sin \phi$

$$\Rightarrow \cos \theta + \cos \phi = a(\sin \theta - \sin \phi)$$

$$\Rightarrow 2 \cos \left(\frac{\theta+\phi}{2}\right) \cos \left(\frac{\theta-\phi}{2}\right) = 2a \sin \left(\frac{\theta-\phi}{2}\right) \cos \left(\frac{\theta+\phi}{2}\right)$$

$$\Rightarrow \cot \left(\frac{\theta-\phi}{2}\right) = a$$

$$\Rightarrow \theta - \phi = 2 \cot^{-1} a$$

$$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2 \cot^{-1} a$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

5. If $y = (\tan x)^x$, then find dy/dx .

Ans. $y = (\tan x)^x$

$$\log y = x \log (\tan x)$$

$$\frac{1}{y} \frac{dy}{dx} = x \left(\frac{\sec^2 x}{\tan x}\right) + \log(\tan x)$$

$$\frac{dy}{dx} = (\tan x)^x \left[\left(\frac{x \sec^2 x}{\tan x}\right) + \log(\tan x)\right]$$

Previous Years' CBSE Board Questions

1.2 Types of Relations

MCQ

1. Let $A = \{3, 5\}$. Then number of reflexive relations on A is
 (a) 2 (b) 4
 (c) 0 (d) 8 (2023)
2. Let R be a relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Then
 (a) $(8, 7) \in R$ (b) $(6, 8) \in R$
 (c) $(3, 8) \in R$ (d) $(2, 4) \in R$ (2023)
3. A relation R is defined on N . Which of the following is the reflexive relation?
 (a) $R = \{(x, y) : x > y, x, y \in N\}$
 (b) $R = \{(x, y) : x + y = 10, x, y \in N\}$
 (c) $R = \{(x, y) : xy \text{ is the square number, } x, y \in N\}$
 (d) $R = \{(x, y) : x + 4y = 10, x, y \in N\}$

(Term I, 2021-22) An

4. The number of equivalence relations in the set $\{1, 2, 3\}$ containing the elements $(1, 2)$ and $(2, 1)$ is
 (a) 0 (b) 1
 (c) 2 (d) 3 (Term I, 2021-22)
5. A relation R is defined on Z as aRb if and only if $a^2 - 7ab + 6b^2 = 0$. Then, R is
 (a) reflexive and symmetric
 (b) symmetric but not reflexive
 (c) transitive but not reflexive
 (d) reflexive but not symmetric

(Term I, 2021-22) Ap

6. Let $A = \{1, 3, 5\}$. Then the number of equivalence relations in A containing $(1, 3)$ is
 (a) 1 (b) 2
 (c) 3 (d) 4 (2020)
7. The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1), (1, 1)\}$ is
 (a) symmetric and transitive, but not reflexive
 (b) reflexive and symmetric, but not transitive
 (c) symmetric, but neither reflexive nor transitive
 (d) an equivalence relation (2020)

VSA (1 mark)

8. Write the smallest reflexive relation on set $A = \{a, b, c\}$. (2021 C)
9. A relation R in a set A is called _____, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$. (2020) R
10. A relation in a set A is called _____ relation, if each element of A is related to itself. (2020) R
11. If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N , write the range of R . (AI 2014)

12. Let $R = \{(a, a^3) : a \text{ is a prime number less than } 5\}$ be a relation. Find the range of R . (Foreign 2014)
13. Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class $[0]$. (Delhi 2014 C)

SA I (2 marks)

14. Check if the relation R in the set \mathbb{R} of real numbers defined as $R = \{(a, b) : a < b\}$ is (i) symmetric, (ii) transitive. (2020)
15. Let W denote the set of words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W \text{ such that } x \text{ and } y \text{ have at least one letter in common}\}$. Show that this relation R is reflexive and symmetric, but not transitive. (2020)

LA I (4 marks)

16. Show that the relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$ is an equivalence relation. (2020)
17. Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive. (2019)
18. Show that the relation R on the set Z of all integers, given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$ is an equivalence relation. (2019)
19. Show that the relation R on \mathbb{R} defined as $R = \{(a, b) : a \leq b\}$, is reflexive and transitive but not symmetric. (NCERT, Delhi 2019)
20. Show that the relation S in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b| \text{ is divisible by } 3\}$ is an equivalence relation. (AI 2019) Ap
21. Let $A = \{1, 2, 3, \dots, 9\}$ and R be the relation in $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$. Prove that R is an equivalence relation. Also obtain the equivalence class $[(2, 5)]$. (Delhi 2014)
22. Let R be a relation defined on the set of natural numbers N as follow:
 $R = \{(x, y) | x \in N, y \in N \text{ and } 2x + y = 24\}$
 Find the domain and range of the relation R . Also, find if R is an equivalence relation or not. (Delhi 2014 C) An

LA II (5/6 marks)

23. If N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by $(a, b) R (c, d)$, if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation. (2023, Delhi 2015)

24. Let $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$, is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2]. (2018)
25. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$ is an equivalence relation. Write all the equivalence classes of R . (AI 2015 C)

1.3 Types of Functions

MCQ

26. The function $f: R \rightarrow R$ defined by $f(x) = 4 + 3 \cos x$ is
 (a) bijective (b) one-one but not onto
 (c) onto but not one-one
 (d) neither one-one nor onto (Term I, 2021-22) (An)
27. The number of functions defined from $\{1, 2, 3, 4, 5\} \rightarrow \{a, b\}$ which are one-one is
 (a) 5 (b) 3
 (c) 2 (d) 0 (Term I, 2021-22)
28. Let $f: R \rightarrow R$ be defined by $f(x) = 1/x$, for all $x \in R$. Then, f is
 (a) one-one (b) onto
 (c) bijective (d) not defined (Term I, 2021-22)
29. The function $f: N \rightarrow N$ is defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

The function f is

- (a) bijective
 (b) one-one but not onto
 (c) onto but not one-one
 (d) neither one-one nor onto

(Term I, 2021-22) (Ev)

VSA (1 mark)

30. If $f = \{(1, 2), (2, 4), (3, 1), (4, k)\}$ is a one-one function from set A to A , where $A = \{1, 2, 3, 4\}$, then find the value of k . (2021 C)

LAI (4 marks)

31. **Case Study :** An organization conducted bike race under two different categories - Boys and girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.



Based on the above information, answer the following questions.

- (i) How many relations are possible from B to G ?
 (ii) Among all the possible relations from B to G , how many functions can be formed from B to G ?
 (iii) Let $R: B \rightarrow B$ be defined by $R = \{(x, y) : x \text{ and } y \text{ are students of the same sex}\}$. Check if R is an equivalence relation.

OR

A function $f: B \rightarrow G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective, justify your answer. (2023) (Ap)

32. Let $f: R - \left\{-\frac{4}{3}\right\} \rightarrow R$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not. (2023)
33. Show that the function $f: (-\infty, 0) \rightarrow (-1, 0)$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in (-\infty, 0)$ is one-one and onto. (2020)

CBSE Sample Questions

1.2 Types of Relations

MCQ

1. A relation R in set $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A ?
 (a) (1, 1) (b) (1, 2) (c) (2, 2) (d) (3, 3)
 (Term I, 2021-22) (An)

2. Let the relation R in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$, given by $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$. Then [1], the equivalence class containing 1, is
 (a) $\{1, 5, 9\}$ (b) $\{0, 1, 2, 5\}$
 (c) ϕ (d) A
 (Term I, 2021-22) (Ev)

VSA (1 mark)

3. How many reflexive relations are possible in a set A whose $n(A) = 3$? (2020-21) (Ap)

4. A relation R in $S = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which element(s) of relation R be removed to make R an equivalence relation? (2020-21)
5. An equivalence relation R in A divides it into equivalence classes A_1, A_2, A_3 . What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$. (2020-21)

SA I (2 marks)

6. Let R be the relation in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$. Show that the relation R is transitive? Write the equivalence class of 0. (2020-21) **Ap**

SA II (3 marks)

7. Check whether the relation R in the set Z of integers defined as $R = \{(a, b) : a + b \text{ is "divisible by 2"}\}$ is reflexive, symmetric or transitive. Write the equivalence class containing 0, i.e., $[0]$. (2020-21)

LA II (5/6 marks)

8. Given a non-empty set X , define the relation R on $P(X)$ as:
For $A, B \in P(X)$, $(A, B) \in R$ iff $A \subset B$. Prove that R is reflexive, transitive, and not symmetric. (2022-23)

9. Define the relation R in the set $N \times N$ as follows:
For $(a, b), (c, d) \in N \times N$, $(a, b) R (c, d)$ iff $ad = bc$. Prove that R is an equivalence relation in $N \times N$. (2022-23)

1.3 Types of Functions

MCQ

10. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B . Based on the given information, f is best defined as
(a) Surjective function (b) Injective function
(c) Bijective function (d) Function (Term I, 2021-22) **Ev**
11. The function $f : R \rightarrow R$ defined as $f(x) = x^3$ is
(a) One-one but not onto
(b) Not one-one but onto
(c) Neither one-one nor onto
(d) One-one and onto (Term I, 2021-22)

VSA (1 mark)

12. Check whether the function $f : R \rightarrow R$ defined as $f(x) = x^3$ is one-one or not. (2020-21)
13. A relation R in the set of real numbers R defined as $R = \{(a, b) : \sqrt{a} = b\}$ is a function or not. Justify (2020-21)

Detailed SOLUTIONS

Previous Years' CBSE Board Questions

1. (b): Total number of reflexive relations on a set having n number of elements $= 2^{n^2 - n}$
Here, $n = 2$
 \therefore Required number of reflexive relations $= 2^{2^2 - 2}$
 $= 2^{4 - 2} = 2^2 = 4$
2. (b): Given, $R = \{(a, b) : a = b - 2, b > 6\}$
Since, $b > 6$, so $(2, 4) \notin R$
Also, $(3, 8) \notin R$ as $3 \neq 8 - 2$
and $(8, 7) \notin R$ as $8 \neq 7 - 2$
Now, for $(6, 8)$, we have
 $8 > 6$ and $6 = 8 - 2$, which is true
 $\therefore (6, 8) \in R$
3. (c): Consider, $R = \{(x, y) : xy \text{ is the square number, } x, y \in N\}$
As, $xx = x^2$, which is the square of natural number x .
 $\Rightarrow (x, x) \in R$. So, R is reflexive.

Concept Applied

A relation R in a set A is called reflexive, if $(a, a) \in R$, for all $a \in A$.

4. (c): Equivalence relations in the set $\{1, 2, 3\}$ containing the elements $(1, 2)$ and $(2, 1)$ are
 $R_1 = \{(1, 2), (2, 1), (1, 1), (2, 2), (3, 3)\}$

$R_2 = \{(1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2), (1, 1), (2, 2), (3, 3)\}$
 \therefore Number of equivalence relations is 2.

Concept Applied

A relation R in a set A is called an equivalence relation, if R is reflexive, symmetric and transitive.

5. (d): Given, $aRb, a, b \in Z$
Reflexive: For $a \in Z$, we have
 $a^2 - 7a - a + 6a^2 = a^2 - 7a^2 + 6a^2 = 0 \Rightarrow (a, a) \in R$
 \therefore Relation is reflexive.
Symmetric: Since, $(6, 1) \in R$
As, $6^2 - 7 \times 6 \times 1 + 6 \times 1^2 = 36 - 42 + 6 = 0$
But $(1, 6) \notin R$. \therefore Relation is not symmetric.
6. (b): Equivalence relations in the set containing the element $(1, 3)$ are
 $R_1 = \{(1, 1), (3, 3), (1, 3), (3, 1), (5, 5)\}$
 $R_2 = \{(1, 1), (3, 3), (5, 5), (1, 5), (5, 1), (3, 5), (5, 3), (1, 3), (3, 1)\}$
 \therefore There are 2 possible equivalence relations.
7. (c): Given $R = \{(1, 2), (2, 1), (1, 1)\}$ is a relation on set $\{1, 2, 3\}$
Reflexive: Clearly $(2, 2), (3, 3) \notin R$
 $\therefore R$ is not a reflexive relation.
Symmetric: Now, $(1, 2) \in R$ and $(2, 1) \in R$. $\therefore R$ is symmetric.
Transitive: Now, $(2, 1) \in R$ and $(1, 2) \in R$ but $(2, 2) \notin R$
 $\therefore R$ is not transitive relation.
 R is symmetric, but neither reflexive nor transitive.

8. We have, $A = \{a, b, c\}$

A relation R on the set A is said to be reflexive if $(a, a) \in R$, $\forall a \in A$

$\therefore R = \{(a, a), (b, b), (c, c)\}$ is the required smallest reflexive relation on A .

9. A relation R in a set A is called symmetric, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

10. A relation in a set A is called reflexive relation, if each element of A is related to itself.

11. Here, $R = \{(x, y) : x + 2y = 8x, y \in N\}$.

For $x = 1, 3, 5, \dots$

$x + 2y = 8$ has no solution in N .

For $x = 2$, we have $2 + 2y = 8 \Rightarrow y = 3$

For $x = 4$, we have $4 + 2y = 8 \Rightarrow y = 2$

For $x = 6$, we have $6 + 2y = 8 \Rightarrow y = 1$

For $x = 8, 10, \dots$

$x + 2y = 8$ has no solution in N .

\therefore Range of $R = \{y : (x, y) \in R\} = \{1, 2, 3\}$

12. Given relation is

$R = \{(a, a^3) : a \text{ is a prime number less than } 5\}$.

$\therefore R = \{(2, 8), (3, 27)\}$. So, the range of R is $\{8, 27\}$.

13. Here, $R = \{(a, b) : 2 \text{ divides } (a - b)\}$

\therefore Equivalence class of $[0] = \{a \in A : (a, 0) \in R\}$.

$\Rightarrow (a - 0)$ is divisible by 2 and $a \in A \Rightarrow a = 0, 2, 4$

Thus $[0] = \{0, 2, 4\}$.

14. We have, $R = \{(a, b) : a < b\}$, where $a, b \in \mathbb{R}$

(i) Symmetric : Let $(x, y) \in R$, i.e., $x < y \Rightarrow x < y$

But $y < x$, so $(x, y) \in R \Rightarrow (y, x) \notin R$

Thus, R is not symmetric.

(ii) Transitive : Let $(x, y), (y, z) \in R$

$\Rightarrow x < y$ and $y < z \Rightarrow x < z$

$\Rightarrow (x, z) \in R$. Thus, R is transitive.

15. We have, $R = \{(x, y) \in W \times W : x \text{ and } y \text{ have at least one letter in common}\}$

Reflexive : Clearly $(x, x) \in R$, because same words will contain all common letters.

$\Rightarrow R$ is reflexive.

Symmetric : Let $(x, y) \in R$ i.e., x and y have at least one letter in common.

$\Rightarrow y$ and x will also have at least one letter in common.

$\Rightarrow (y, x) \in R$

$\Rightarrow R$ is symmetric.

Transitive : Let, $x = \text{LAND}$, $y = \text{NOT}$ and $z = \text{HOT}$

Clearly $(x, y) \in R$ as x and y have a common letter and $(y, z) \in R$ as y and z have 2 common letters.

but $(x, z) \notin R$ as x and z have no letter in common.

Hence, R is not transitive.

Concept Applied

\Rightarrow A relation R in a set A is not transitive if for $(a, b) \in R$ and $(b, c) \in R$ but $(a, c) \notin R$

16. We have, $A = \{1, 2, 3, 4, 5, 6\}$ and $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$

(i) Reflexive : For any $a \in A$

$|a - a| = 0$, which is divisible by 2.

Thus, $(a, a) \in R$. So, R is reflexive.

(ii) Symmetric : For any $a, b \in A$

Let $(a, b) \in R$

$\Rightarrow |a - b|$ is divisible by 2 $\Rightarrow |b - a|$ is divisible by 2

$\Rightarrow (b, a) \in R \therefore (a, b) \in R \Rightarrow (b, a) \in R \therefore R$ is symmetric.

(iii) Transitive : For any $a, b, c \in A$

Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow |a - b|$ is divisible by 2 and $|b - c|$ is divisible by 2.

$\Rightarrow a - b = \pm 2k_1$ and $b - c = \pm 2k_2 \forall k_1, k_2 \in N$

$\Rightarrow a - b + b - c = \pm 2(k_1 + k_2) \Rightarrow a - c = \pm 2k_3 \forall k_3 \in N$

$\Rightarrow |a - c|$ is divisible by 2 $\Rightarrow (a, c) \in R \therefore R$ is transitive.

Hence, R is an equivalence relation.

17. We have, $A = \{1, 2, 3, 4, 5, 6\}$ and a relation R on A defined as $R = \{(a, b) : b = a + 1\}$

Reflexive : Let $(a, a) \in R$

$\Rightarrow a = a + 1 \Rightarrow a - a = 1 \Rightarrow 0 = 1$, which is not possible.

$\therefore (a, a) \notin R \Rightarrow R$ is not reflexive.

Symmetric : Let $(a, b) \in R \Rightarrow b = a + 1$... (i)

Now, if $(b, a) \in R$

$\Rightarrow a = b + 1 \Rightarrow b = b + 1 + 1$ (using (i))

$\Rightarrow b = b + 2 \Rightarrow b - b = 2 \Rightarrow 0 = 2$, which is not possible

$\Rightarrow (b, a) \notin R \Rightarrow R$ is not symmetric.

Transitive : Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow b = a + 1$ and $c = b + 1 \Rightarrow c = a + 1 + 1$

$\Rightarrow c = a + 2 \neq a + 1 \Rightarrow (a, c) \notin R \Rightarrow R$ is not transitive.

18. We have, $R = \{(a, b) : 2 \text{ divides } (a - b)\}$

Reflexive : For any $a \in Z$, $a - a = 0$ and 2 divides 0.

$\Rightarrow (a, a) \in R$ for every $a \in Z \therefore R$ is a reflexive.

Symmetric : Let $(a, b) \in R$

$\Rightarrow 2$ divides $(a - b)$

$\Rightarrow a - b = 2m$, for some $m \in Z$

$\Rightarrow b - a = 2m$

$\Rightarrow 2$ divides $b - a$

$\Rightarrow (b, a) \in R$

$\therefore R$ is symmetric.

Transitive : Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow 2$ divides $(a - b)$ and 2 divides $(b - c)$

$\Rightarrow a - b = 2m$ and $b - c = 2n$ for some $m, n \in Z$

$\Rightarrow a - b + b - c = 2m + 2n$

$\Rightarrow a - c = 2(m + n)$

$\Rightarrow 2$ divides $a - c$

$\Rightarrow (a, c) \in R$

$\therefore R$ is transitive.

Hence, R is an equivalence relation.

19. We have, $R = \{(a, b) : a \leq b, a, b \in \mathbb{R}\}$

(i) Reflexive : Since $a \leq a \therefore aRa \forall a \in \mathbb{R}$

Hence, R is reflexive.

(ii) Symmetric : $(a, b) \in R$ such that $aRb \Rightarrow a \leq b \not\Rightarrow b \leq a$

So, $(b, a) \notin R$.

Hence, R is not symmetric.

(iii) Transitive : Let $a, b, c \in \mathbb{R}$ such that aRb and bRc

Now, $aRb \Rightarrow a \leq b$... (i) and $bRc \Rightarrow b \leq c$... (ii)

From (i) and (ii), we have $a \leq b \leq c \Rightarrow a \leq c \therefore aRc$

Hence, relation R is transitive.

20. We have, $A = \{x \in Z : 0 \leq x \leq 12\}$

$\therefore A = \{0, 1, 2, 3, \dots, 12\}$

Also, $S = \{(a, b) : a, b \in Z, |a - b| \text{ is divisible by } 3\}$

(i) Reflexive : For any $a \in A$,
 $|a - a| = 0$, which is divisible by 3

Thus, $(a, a) \in S \therefore S$ is reflexive.

(ii) Symmetric : Let $(a, b) \in S$

$\Rightarrow |a - b|$ is divisible by 3.

$\Rightarrow |b - a|$ is divisible by 3 $\Rightarrow (b, a) \in S$ i.e. $(a, b) \in S \Rightarrow (b, a) \in S$

$\therefore S$ is symmetric.

(iii) Transitive :

Let $(a, b) \in S$ and $(b, c) \in S$

$\Rightarrow |a - b|$ is divisible by 3 and $|b - c|$ is divisible by 3.

$\Rightarrow (a - b) = \pm 3k_1$ and $(b - c) = \pm 3k_2; \forall k_1, k_2 \in N$

$\Rightarrow (a - b) + (b - c) = \pm 3(k_1 + k_2)$

$\Rightarrow (a - c) = \pm 3(k_1 + k_2); \forall k_1, k_2 \in N$

$\Rightarrow |a - c|$ is divisible by 3 $\Rightarrow (a, c) \in S \therefore S$ is Transitive.

Hence, S is an equivalence relation.

Concept Applied

\Rightarrow A relation R in a set A is called

- (i) reflexive, if $(a, a) \in R$, for all $a \in A$
- (ii) symmetric, if $(a, b) \in R \Rightarrow (b, a) \in R$, for all $a, b \in A$
- (iii) transitive, if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$, for all $a, b, c \in A$

21. Given $A = \{1, 2, 3, 4, \dots, 9\}$

To show : R is an equivalence relation.

(i) Reflexive : Let (a, b) be an arbitrary element of $A \times A$.

Then, we have $(a, b) \in A \times A \Rightarrow a, b \in A$

$\Rightarrow a + b = b + a$ (by commutativity of addition on $A \subset N$)

$\Rightarrow (a, b) R (a, b)$

Thus, $(a, b) R (a, b)$ for all $(a, b) \in A \times A$. So, R is reflexive.

(ii) Symmetric : Let $(a, b), (c, d) \in A \times A$ such that $(a, b) R (c, d)$

$\Rightarrow a + d = b + c \Rightarrow b + c = a + d$

$\Rightarrow c + b = d + a$ (by commutativity of addition on $A \subset N$)

$\Rightarrow (c, d) R (a, b)$.

Thus, $(a, b) R (c, d) \Rightarrow (c, d) R (a, b)$ for all $(a, b), (c, d) \in A \times A$.

So, R is symmetric.

(iii) Transitive : Let $(a, b), (c, d), (e, f) \in A \times A$ such that $(a, b) R (c, d)$ and $(c, d) R (e, f)$

Now, $(a, b) R (c, d) \Rightarrow a + d = b + c$... (i)

and $(c, d) R (e, f) \Rightarrow c + f = d + e$... (ii)

Adding (i) and (ii), we get $(a + d) + (c + f) = (b + c) + (d + e)$

$\Rightarrow a + f = b + e \Rightarrow (a, b) R (e, f)$

Thus, $(a, b) R (c, d)$ and $(c, d) R (e, f) \Rightarrow (a, b) R (e, f)$.

So, R is transitive. $\therefore R$ is an equivalence relation.

Equivalence class of $\{(2, 5)\} = \{(x, y) \in N \times N : (x, y) R (2, 5)\}$

$= \{(x, y) \in N \times N : x + 5 = y + 2\}$

$= \{(x, y) \in N \times N : y = x + 3\} = \{(x, x + 3) : x \in A\}$

$= \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$.

Answer Tips

\Rightarrow First, prove the given relation is an equivalence relation and then find the equivalence class by using the given relation.

22. Here, $R = \{(x, y) \mid x \in N, y \in N \text{ and } 2x + y = 24\}$

$R = \{(1, 22), (2, 20), (3, 18), \dots, (11, 2)\}$

Domain of $R = \{1, 2, 3, 4, \dots, 11\}$

Range of $R = \{2, 4, 6, 8, 10, 12, \dots, 22\}$

R is not reflexive as if $(2, 2) \in R \Rightarrow 2 \times 2 + 2 = 6 \neq 24$

In fact R is neither symmetric nor transitive.

$\Rightarrow R$ is not an equivalence relation.

23. (i) Reflexive : Let (a, b) be an arbitrary element of $N \times N$. Then, $(a, b) \in N \times N$

$\Rightarrow ab(b + a) = ba(a + b)$

[by commutativity of addition and multiplication on N]

$\Rightarrow (a, b) R (a, b)$

So, R is reflexive on $N \times N$.

(ii) Symmetric : Let $(a, b), (c, d) \in N \times N$ such that

$(a, b) R (c, d)$.

$\Rightarrow ad(b + c) = bc(a + d) \Rightarrow cb(d + a) = da(c + b)$

[by commutativity of addition and multiplication on N]

Thus, $(a, b) R (c, d) \Rightarrow (c, d) R (a, b)$ for all $(a, b), (c, d) \in N \times N$.

So, R is symmetric on $N \times N$.

(iii) Transitive : Let $(a, b), (c, d), (e, f) \in N \times N$ such that

$(a, b) R (c, d)$ and $(c, d) R (e, f)$. Then,

$(a, b) R (c, d) \Rightarrow ad(b + c) = bc(a + d)$

$\Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad} \Rightarrow \frac{1}{b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{d}$... (i)

and $(c, d) R (e, f) \Rightarrow cf(d + e) = de(c + f)$

$\Rightarrow \frac{d+e}{de} = \frac{c+f}{cf} \Rightarrow \frac{1}{d} + \frac{1}{e} = \frac{1}{c} + \frac{1}{f}$... (ii)

Adding (i) and (ii), we get

$\left(\frac{1}{b} + \frac{1}{c}\right) + \left(\frac{1}{d} + \frac{1}{e}\right) = \left(\frac{1}{a} + \frac{1}{d}\right) + \left(\frac{1}{c} + \frac{1}{f}\right)$

$\Rightarrow \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f} \Rightarrow \frac{b+e}{be} = \frac{a+f}{af}$

$\Rightarrow af(b + e) = be(a + f) \Rightarrow (a, b) R (e, f)$

So, R is transitive on $N \times N$.

Hence, R is an equivalence relation.

24. We have, $A = \{x \in Z : 0 \leq x \leq 12\}$

$\therefore A = \{0, 1, 2, 3, \dots, 12\}$

and $S = \{(a, b) \mid |a - b| \text{ is divisible by } 4\}$

(i) Reflexive : For any $a \in A$, $|a - a| = 0$, which is divisible by 4. Thus, $(a, a) \in R \therefore R$ is reflexive.

(ii) Symmetric : Let $(a, b) \in R$

$\Rightarrow |a - b|$ is divisible by 4

$\Rightarrow |b - a|$ is divisible by 4 $\Rightarrow (b, a) \in R$

i.e., $(a, b) \in R \Rightarrow (b, a) \in R \therefore R$ is symmetric.

(iii) Transitive : Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow |a - b|$ is divisible by 4 and $|b - c|$ is divisible by 4

$\Rightarrow a - b = \pm 4k_1$ and $b - c = \pm 4k_2; \forall k_1, k_2 \in N$

$\Rightarrow (a - b) + (b - c) = \pm 4(k_1 + k_2); \forall k_1, k_2 \in N$

$\Rightarrow a - c = \pm 4(k_1 + k_2); \forall k_1, k_2 \in N$

$\Rightarrow |a - c|$ is divisible by 4 $\Rightarrow (a, c) \in R \therefore R$ is transitive.

Hence, R is an equivalence relation.

The set of elements related to 1 is $\{1, 5, 9\}$.

Equivalence class for $[2]$ is $\{2, 6, 10\}$.

Concept Applied

\Rightarrow In a relation R in a set A , the set of all elements related to any element $a \in A$ is denoted by $[a]$

i.e., $[a] = \{x \in A : (x, a) \in R\}$

Here, $[a]$ is called an equivalence class of $a \in A$.

25. We have, $A = \{1, 2, 3, 4, 5\}$
and $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$

(i) Reflexive : For any $a \in A$,
 $|a - a| = 0$, which is divisible by 2

Thus, $(a, a) \in R \therefore R$ is reflexive.

(ii) Symmetric : Let $(a, b) \in R$

$\Rightarrow |a - b|$ is divisible by 2

$\Rightarrow |b - a|$ is divisible by 2 $\Rightarrow (b, a) \in R$

i.e., $(a, b) \in R \Rightarrow (b, a) \in R \therefore R$ is symmetric.

(iii) Transitive : Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow |a - b|$ is divisible by 2 and $|b - c|$ is divisible by 2

$\Rightarrow a - b = \pm 2k_1$ and $b - c = \pm 2k_2; \forall k_1, k_2 \in \mathbb{N}$

$\Rightarrow (a - b) + (b - c) = \pm 2(k_1 + k_2); \forall k_1, k_2 \in \mathbb{N}$

$\Rightarrow (a - c) = \pm 2(k_1 + k_2); \forall k_1, k_2 \in \mathbb{N}$

$\Rightarrow |a - c|$ is divisible by 2 $\Rightarrow (a, c) \in R \therefore R$ is transitive.

Hence, R is an equivalence relation.

Further R has only two equivalence classes, namely $[1] = [3] = [5] = \{1, 3, 5\}$ and $[2] = [4] = [2, 4]$.

26. (d): We have, $f(x) = 4 + 3 \cos x, \forall x \in \mathbb{R}$

At $x = \frac{\pi}{2}, f\left(\frac{\pi}{2}\right) = 4 + 3 \cos \frac{\pi}{2} = 4 \Rightarrow f\left(-\frac{\pi}{2}\right) = 4 + 3 \cos\left(-\frac{\pi}{2}\right) = 4$

Since, $f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$, But $\frac{\pi}{2} \neq -\frac{\pi}{2}$

Therefore, f is not one-one.

As $-1 \leq \cos x \leq 1, \forall x \in \mathbb{R} \Rightarrow 1 \leq 4 + 3 \cos x \leq 7, \forall x \in \mathbb{R}$

$\Rightarrow f(x) \in [1, 7]$, where $[1, 7]$ is subset of $\mathbb{R} \therefore f$ is not onto.

Concept Applied

\Rightarrow Range of $\cos x$ is $[-1, 1]$.

27. (d): $\because f : X \rightarrow Y$ is one-one, if different element of X have different image in Y under f . But here, no such situation is possible.

28. (d): Given $f(x) = \frac{1}{x}$, for all $x \in \mathbb{R}$

At $x = 0 \in \mathbb{R}, f(x)$ is not defined.

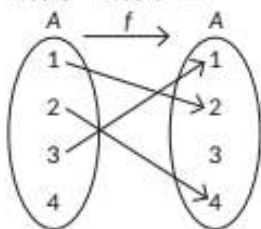
29. (c): Given, $f(x) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

Now, $f(1) = \frac{1+1}{2} = 1, f(2) = \frac{2}{2} = 1$

$\Rightarrow f(1) = f(2)$ but $1 \neq 2 \therefore f$ is not one-one.

But f is onto (\because range of f is \mathbb{N} .)

30. We have, $A = \{1, 2, 3, 4\}$ function $f : A \rightarrow A$ is one-one and $f(1) = 2, f(2) = 4, f(3) = 1, f(4) = k$



As f is one-one, so no two element of A has same image in A .

$\therefore f(4) = 3 \Rightarrow k = 3$

Concept Applied

\Rightarrow For a function to be one-one, no two elements should have the same image in A .

31. (i) Here $n(B) = 3$ and $n(G) = 2$

\therefore Number of relation from B to $G = 2^{3 \times 2} = 2^6$

(ii) Number of functions formed from B to $G = 2^3 = 8$

(iii) We have, $R = \{(x, y) = x \text{ and } y \text{ are students of the same sex}\}$

$\therefore R$ is reflexive as $(x, x) \in R$.

R is symmetric as $(x, y) \in R \Rightarrow (y, x) \in R$.

Since, $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$

Hence, R is an equivalence relations.

OR

We have $f : B \rightarrow G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$

Since, elements b_1 and b_3 have the same image, therefore, the functions is not one-one but it is many one functions.

Since, every element in G has its pre-image in B , so the functions is onto.

For bijection, function should be one-one and onto both.

Hence, the function is surjective but not injective.

32. The function $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathbb{R}$ is given by $f(x) = \frac{4x}{3x+4}$.

One-one : Let $x, y \in \mathbb{R} - \left\{-\frac{4}{3}\right\}$ such that $f(x) = f(y)$

$$\Rightarrow \frac{4x}{3x+4} = \frac{4y}{3y+4}$$

$$\Rightarrow 4x(3y+4) = 4y(3x+4) \Rightarrow 12xy + 16x = 12xy + 16y$$

$$\Rightarrow 16x = 16y \Rightarrow x = y$$

$\therefore f$ is one-one.

Onto : Let y be an arbitrary element of \mathbb{R} . Then $f(x) = y$

$$\Rightarrow \frac{4x}{3x+4} = y \Rightarrow 4x = 3xy + 4y \Rightarrow 4x - 3xy = 4y \Rightarrow x = \frac{4y}{4-3y}$$

$$\text{As } y \in \mathbb{R} - \left\{\frac{4}{3}\right\}, \frac{4y}{4-3y} \in \mathbb{R}$$

$$\text{Also, } \frac{4y}{4-3y} \neq -\frac{4}{3} \text{ as if}$$

$$\frac{4y}{4-3y} = -\frac{4}{3} \Rightarrow 12y = 12y - 16, \text{ which is not possible.}$$

Thus, $x = \frac{4y}{4-3y} \in \mathbb{R} - \left\{-\frac{4}{3}\right\}$ such that

$$f(x) = f\left(\frac{4y}{4-3y}\right) = \frac{4\left(\frac{4y}{4-3y}\right)}{3\left(\frac{4y}{4-3y}\right)+4} = \frac{16y}{12y+16-12y} = \frac{16y}{16} = y$$

So, every element in $\mathbb{R} - \left\{\frac{4}{3}\right\}$ has pre-image in $\mathbb{R} - \left\{-\frac{4}{3}\right\}$

$\therefore f$ is not onto.

33. Given, $f(x) = \frac{x}{1+|x|}, x \in (-\infty, 0)$

$$= \frac{x}{1-x}$$

($\because x \in (-\infty, 0), |x| = -x$)

For one-one : Let $f(x_1) = f(x_2)$, $x_1, x_2 \in (-\infty, 0)$

$$\Rightarrow \frac{x_1}{1-x_1} = \frac{x_2}{1-x_2} \Rightarrow x_1(1-x_2) = x_2(1-x_1)$$

$$\Rightarrow x_1 - x_1x_2 = x_2 - x_1x_2 \Rightarrow x_1 = x_2$$

Thus, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$\therefore f$ is one-one

For onto : Let $f(x) = y$

$$\Rightarrow y = \frac{x}{1-x} \Rightarrow y(1-x) = x \Rightarrow y - xy = x$$

$$\Rightarrow x + xy = y \Rightarrow x(1+y) = y \Rightarrow x = \frac{y}{1+y}$$

Here, $y \in (-1, 0)$

So, x is defined for all values of y in codomain. $\therefore f$ is onto.

Concept Applied

\Rightarrow A function $f: A \rightarrow B$ is called

(i) one-one or injective function, if distinct elements of A have distinct images in B .

i.e., for $a, b \in A$, $f(a) = f(b) \Rightarrow a = b$

(ii) onto or surjective function, if for every element $b \in B$, there exists some $a \in A$ such that $f(a) = b$.

CBSE Sample Questions

1. (b): We have, $(1, 2) \in R$ but $(2, 1) \notin R$

So, $(1, 2)$ should be removed from R to make it an equivalence relation. (1)

2. (a): We have, $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$

\therefore The set of elements related to 1 is $\{1, 5, 9\}$.

So, equivalence class for $\{1\}$ is $\{1, 5, 9\}$ (1)

3. Number of reflexive relations on a set having n elements = $2^{n(n-1)}$

So, required number of reflexive relations = $2^{3(3-1)} = 2^6$ (1)

4. We have, $R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$ which is reflexive and transitive.

For R to be symmetric $(1, 2)$ should be removed from R . (1)

5. As we know that, union of all equivalence classes of a set is the set itself.

$$\therefore A_1 \cup A_2 \cup A_3 = A$$

$$\text{Also, } A_1 \cap A_2 \cap A_3 = \phi$$

\therefore Equivalence classes are either equal or disjoint (1)

6. Let $(a, b) \in R$ and $(b, c) \in R$. Then, 2 divides $(a - b)$ and 2 divides $(b - c)$: where $a, b, c \in Z$

So, 2 divides $[(a - b) + (b - c)]$

\Rightarrow 2 divides $(a - c) \Rightarrow (a, c) \in R$. So, relation R is transitive. (1)

Equivalence class of 0 = $\{0, \pm 2, \pm 4, \pm 6, \dots\}$ (1)

7. (i) Reflexive : Since, $a + a = 2a$ which is even.

$$\therefore (a, a) \in R \forall a \in Z$$

Hence, R is reflexive. (1/2)

(ii) Symmetric : If $(a, b) \in R$, then $a + b = 2\lambda \Rightarrow b + a = 2\lambda$

$\Rightarrow (b, a) \in R$. Hence, R is symmetric. (1)

(iii) Transitive : If $(a, b) \in R$ and $(b, c) \in R$

then $a + b = 2\lambda$ (i) and $b + c = 2\mu$ (ii)

Adding (i) and (ii), we get

$$a + 2b + c = 2(\lambda + \mu) \Rightarrow a + c = 2(\lambda + \mu - b)$$

$$\Rightarrow a + c = 2k, \text{ where } k = \lambda + \mu - b \Rightarrow (a, c) \in R$$

Hence, R is transitive. (1)

Equivalence class containing 0 i.e.,

$$[0] = \{\dots, -4, -2, 0, 2, 4, \dots\} \quad (1/2)$$

8. We have, a relation R on X such that, $(A, B) \in R$ iff $A \subset B$ for $A, B \in P(X)$. (1/2)

Reflexive : Clearly every set is a subset of itself.

$$\Rightarrow (A, A) \in R$$

$\therefore R$ is reflexive. (1)

Symmetric : Let $(A, B) \in R$

$$\Rightarrow A \subset B$$

$\Rightarrow B$ is a super set of A .

$$\Rightarrow B \not\subset A \Rightarrow (B, A) \notin R \quad (1/2)$$

$\therefore R$ is not symmetric. (1)

Transitive : Let $(A, B) \in R$ and $(B, C) \in R$, for all $A, B, C \in P(X)$

$$\Rightarrow A \subset B \text{ and } B \subset C \Rightarrow A \subset B \subset C \quad (1/2)$$

$$\Rightarrow A \subset C \Rightarrow (A, C) \in R$$

$\therefore R$ is transitive. (1)

Hence, R is reflexive and transitive but not symmetric. (1/2)

9. Reflexive : Let $(a, b) \in N \times N$. Then $ab = ba$

(By commutativity of multiplication of natural number)

$$\Rightarrow (a, b) R (b, a)$$

Thus, $(a, b) R (b, a)$ for all $(a, b) \in N \times N$

So, R is reflexive. (1)

Symmetric : Let $(a, b), (c, d) \in N \times N$ such that $(a, b) R (c, d)$

$$\Rightarrow ad = bc \Rightarrow bc = ad \Rightarrow cb = da$$

(By commutativity of multiplication of natural numbers)

$$\Rightarrow (c, d) R (a, b)$$

Thus, $(a, b) R (c, d) = (c, d) R (a, b)$ for $(a, b), (c, d) \in N \times N$

So, R is symmetric. (1)

Transitive : Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b) R (c, d)$ and $(c, d) R (e, f)$

$$\text{Now, } (a, b) R (c, d) \Rightarrow ad = bc \quad \dots(i)$$

$$\text{and } (c, d) R (e, f) \Rightarrow cf = de \quad \dots(ii)$$

Multiplying (i) and (ii), we get $ad \cdot cf = bc \cdot de$ (1)

$$\Rightarrow af = be \Rightarrow (a, b) R (e, f)$$

Thus, $(a, b) R (c, d)$ and $(c, d) R (e, f) \Rightarrow (a, b) R (e, f)$ (1)

So, R is transitive.

$\therefore R$ is an equivalence relation. (1)

10. (b): As every pre-image $x \in A$, has a unique image $y \in B$.

$\Rightarrow f$ is injective function. (1)

11. (d): Let $x_1, x_2 \in R$ be such that $f(x_1) = f(x_2)$

$$\Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2 \Rightarrow f \text{ is one-one.}$$

Let $f(x) = x^3 = y$ for some arbitrary element $y \in R \Rightarrow x = y^{1/3}$

$$\Rightarrow f(y^{1/3}) = y$$

Every image $y \in R$ has a unique pre-image in R .

$\Rightarrow f$ is onto

$\therefore f$ is one-one and onto. (1)

12. Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$.

$$\Rightarrow (x_1)^3 = (x_2)^3$$

$\Rightarrow x_1 = x_2$, hence $f(x)$ is one-one. (1)

13. Since \sqrt{a} is not defined for $a \in (-\infty, 0)$

$\therefore R = \{(a, b) : \sqrt{a} = b\}$ is not a function. (1)